Letters

Assumption of homogeneous elasticity in theories of lamellar texture of polymers

In a recent paper by Owen and Ward [1] a model to be referred to as (I) is proposed to explain the mechanical anisotropy in specially oriented sheets of low density polyethylene. The model consists of stacked platelets or "laths" which have infinite extent in one direction only. The platelets are assumed to be rigid and to be separated by incompressible material of shear modulus G. These assumptions are exactly the same as in a companion paper by Davies *et al* [2] except that, in the latter, the platelets were assumed to be of infinite extent in *both* lateral dimensions. We refer to this model as (II).

It is pointed out in [1] that model I is less restrictive than model II in that assuming the platelets of infinite extent in one direction only allows *pure* shear deformations to take place, only *simple* shear being possible for (II).

By assuming an extension e along the normal to the lamellae and $-e$ perpendicular to it, the state of pure shear leads to a Young's modulus perpendicular to the planes of the lamellae of $E = 4\ G$. Now while this is certainly true for a state of *homogeneous* shear, the assumption that such homogeneous shear is present in the material between the lamellae leads to inconsistencies, as we shall show.

If the lamellae are assumed to be rigid, then a strain $-e$ parallel to their planes cannot exist at the boundaries of the amorphous region where, however, physical continuity must exist. Thus the conditions for a state of homogeneous strain in model I cannot be obtained unless the "laths" are infinitely narrow (of course if the condition of rigidity of the lamellae is also relaxed, they could then contract to satisfy the boundary conditions).

The state of affairs in I would seem to be more correctly described by a model investigated in 1958 by Gent and Lindley [3, 4] with reference to bonded rubber blocks in tension and compression, the blocks being bonded to *rigid end plates*.

For a rectangular block of infinite extent in one direction, Gent and Lindley showed that the Young's modulus perpendicular to the bonded face was given by

$$E = \frac{4E_0}{3} + \frac{E_0}{3}\frac{W^2}{t^2} = 4G + \frac{GW^2}{t^2}$$

where W is the width and t the thickness of the block, $E_0 = 3\ G$ = Young's modulus of the rubber.

Assuming that this model applies, we see that only if $t \gg W$ will the Owen and Ward result of $E = 4\ G$ apply. Now there are very few measurements of t or W for real lamellae in polymers, but Cowking and Rider [5], from experimental results, suggested a value for t of 85 Å. If $W/t = 1$ then $E = 5\ G$ and the lamella width W would also be 85 Å or about 11 unit cell widths in the a-direction for polyethylene. Such a small lamella width should evidence itself by line broadening in wide angle X-ray diffraction. If $W/t = 10$, however, E becomes $104\ G$ whereas for $W/t = 100$ it is $10000\ G$. Thus it appears that for the model of rigid lamellae separated by amorphous incompressible material, the deformation perpendicular to the plane of the lamellae will be negligible unless the lamellae are of very small lateral dimensions, less than or equal to the thickness of the amorphous layer.

The measured figure for E of 1.5 kbar found by Owen and Ward and the evidence from Keller and Pope [6] of appreciable compliance perpendicular to the planes of the lamellae must imply either very narrow lamellae (akin to fibrils perhaps) or a model of very different properties to the one assumed by Owen and Ward. A more detailed account of the theory of lamella composites will appear elsewhere.

References

1. A. J. OWEN and I. M. WARD, *J. Mater. Sci.* **6** (1971) 485.
2. G. R. DAVIES, A. J. OWEN, I. M. WARD and V. B. GUPTA, *J. Macromol. Sci-Phys B* **6** (1972) 215.
3. A. N. GENT and P. B. LINDLEY, *Proc. I. Mech. E.* **173** (1959) 111.
4. *Idem, Proc. Roy. Soc. Lond. A* **249** (1959) 195.
5. A. COWKING and J. G. RIDER, *J. Mater. Sci.* **4** (1969) 1051.
6. A. KELLER and D. P. POPE, *ibid* **6** (1971) 453.

Received 25 July
and accepted 24 August 1973

R. G. C. ARRIDGE
H. H. Wills Physics Laboratory,
University of Bristol, UK